As at the inlet ($z = 0$) we have

$$T = T_1, \quad \theta(0) = 0,$$

the solution of equation (A7) is

$$\theta = 1 - e^{-z},$$

or

$$T = T_s + (T_1 - T_s)\,e^{-z},$$

that is to say the temperature difference between the bulk and the wall is decaying exponentially, as suggested by the full two-dimensional analysis.

Int. J. Heat Mass Transfer. Vol. 27, No. 8, pp. 1417–1418, 1984
Printed in Great Britain

0017-9310/84 \$3.00 + 0.00
Pergamon Press Ltd.

CONCENTRATION PROFILES NEAR A WALL AT $Pr \gg 1$

B. A. KADER

Institute of Atmospheric Physics, U.S.S.R. Academy of Sciences, Moscow, U.S.S.R.

(*Received* 21 *January* 1983 *and in revised form* 9 *June* 1983)

NOMENCLATURE

c concentration
d tube diameter
j mass flux
Pr Prandtl number
Re Reynolds number
u_* friction velocity
y coordinate measured normal to a wall.

Greek symbols

δ_h molecular diffusivity sublayer thickness
λ skin friction coefficient
ν kinematic viscosity.

Subscripts

w wall quantities
o outer layer quantities
$+$ dimensionless quantities.

IN REF. [1], an equation has been suggested for the profiles of a passive, i.e. not affecting the dynamics of flow, admixture, which is valid at any value of the molecular Prandtl number. The derivation of this equation was based on interpolation between the limiting laws for temperature or concentration distribution at $Pr \ll 1$ and $Pr \gg 1$. The validity of the law referring to very small Pr numbers is easily verified, since the available literature is abundant in experimental data on temperature profiles in liquid metals where $Pr \ll 1$. The situation with the other limiting case of $Pr \gg 1$ is quite different. The experimental data on heat transfer in liquids of high viscosity are confined to the values of $Pr \lesssim 10^2$, and the accuracy of these data is rather low with the increase of Pr because the heat flux must be very small to justify the treatment of heat as a passive impurity. True, the measurements of concentration profiles of material admixtures in liquids do not require the smallness of the flux and are characterized, as a rule, by $Pr \gtrsim 10^3$, but the data of such measurements have been virtually nonexistent in available literature until recently. The reasons for this are quite obvious: at $Pr \gg 1$ the main concentration drop takes place within a very thin sublayer near a wall, the measurements of which present a tremendous experimental problem. Thus, according to ref. [1], the thickness of the molecular diffusivity sublayer, where more than two-thirds of the total concentration drop is achieved, can be estimated as $\delta_h \approx 12/(Re\,Pr^{1/3}\sqrt{(\lambda/8)})$. Therefore, e.g. in a tube of diameter $d = 20$ mm, δ_h is only about 0.05 mm at $Re = 10^4$ and $Pr = 10^3$.

However, recently data have been published [2] that refer to the measurements of concentration profiles $c(y)$ by the electrochemical method (which does not affect the wall smoothness) at $Pr = 1320$ and $Pr = 2140$ and for the distances y from the wall beginning with y-values of the order of 10 μm. In ref. [2] a photometric method of concentration measurements was used, which is based on the visualization of the surface of fixed concentration with the aid of special additives and colour tracers. It seems worthwhile to compare the general equation proposed in ref. [1] for the evaluation of the concentration profile at any Pr with these new data.

In Fig. 1, the solid line represents this general equation, which in the case of large Pr and $y_+ \gg Pr^{-3}$ takes on the form

$$c^+ = \frac{c - c_w}{c_o - c_w} = 0.068 Pr^{1/3}\, y_+ \exp(-\Gamma) + \exp(-1/\Gamma),$$

$$\Gamma = 2 \times 10^{-3} Pr\, y_+, \quad y_+ = y u_*/\nu. \quad (1)$$

The dashed curve corresponds to the formula

$$c^+ = \frac{\sqrt{3}}{2\pi}\left[\frac{1}{2}\ln\frac{(y_+ + A)^2}{y_+^2 - Ay_+ + A^2} + \sqrt{3}\arctan\frac{2y_+ - A}{A\sqrt{3}} + \frac{\pi}{2\sqrt{3}}\right], \quad A = (b\,Pr)^{-1/3}, \quad (2)$$

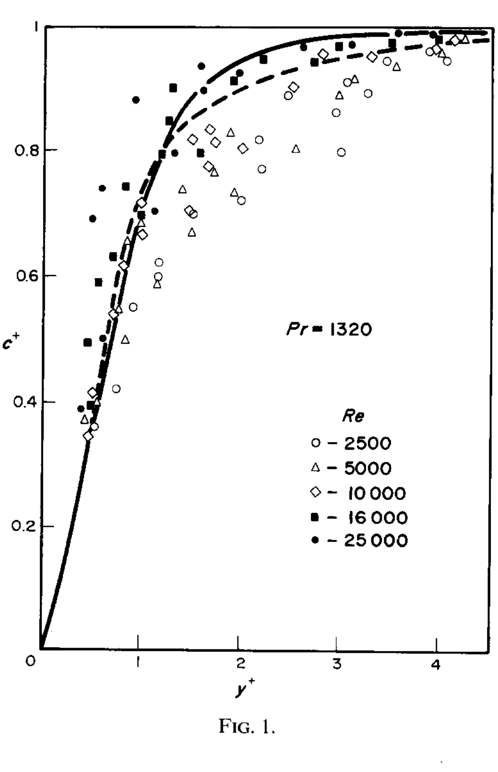

FIG. 1.

which can be obtained by integration of the well-known equation

$$\frac{\partial c_+}{\partial y_+} = \frac{1}{Pr^{-1} + by_+^n}, \quad c_+ = \frac{(c-c_w)u_*}{j_w}, \tag{3}$$

where in accordance with the experimental data from ref. [2] it is assumed that $n = 3$. The coefficient b in equation (3) is assumed to be equal to 1/1320; this value of b lies within the limits for b-values determined in ref. [2] for $Re \gtrsim 16 \times 10^3$ and it leads to $A = 1$, which simplifies the calculations by formula (2).

A large spread of the experimental points in Fig. 1 apparently indicates that the turbulent structure in the viscous sublayer depends on Re in the studied range of small Reynolds numbers. This dependence is especially noticeable at $Re \lesssim 10^4$, but it is not negligible up to the maximum values of Re reached in the experiments. These conclusions agree with the results of the temperature profile measurements presented in ref. [3], where it was found that for $Re \lesssim 25 \times 10^3$ the additive term in the logarithmic temperature distribution law, which represents the temperature drop in a viscous sublayer, depends not only on Pr, but also on Re. The presented data also show that the values of Re, at which the concentration profile can be considered as stabilized, were not attained in the considered experiments. This remark is even more valid for the $c(y)$ profiles measured in ref. [2] at $Pr = 2140$ and $Re \leqslant 10^4$, which for this very reason, are not considered here.

Figure 1 shows also that at $Re \gtrsim 20 \times 10^3$ the measured concentration profile is satisfactorily described both by the limiting relation (1) and equation (2). It should be noted, however, that best agreement is obtained at $Re = 16 \times 10^3$, while at $Re = 25 \times 10^3$ the agreement is somewhat worse.

REFERENCES

1. B. A. Kader, Temperature and concentration profiles in fully turbulent boundary layers, *Int. J. Heat Mass Transfer* **24**, 1541–1544 (1981).
2. J. F. Flender and J. W. Hiby, Investigation of solid–liquid mass transfer by a photometric method, *Germ. Chem. Engng* **4**, 370–379 (1981).
3. R. A. Gowen and J. W. Smith, The effect of the Prandtl number on temperature profiles for heat transfer in turbulent pipe flow, *Chem. Engng Sci.* **22**, 1701–1711 (1967).

Int. J. Heat Mass Transfer. Vol. 27, No. 8, pp. 1418–1421, 1984
Printed in Great Britain
0017-9310/84 $3.00 + 0.00

ON A SLOWLY EVAPORATING DROPLET NEAR A HOT PLATE

Asok K. Sen† and C. K. Law
Department of Mechanical and Nuclear Engineering, Northwestern University, Evanston, IL 60201, U.S.A.

(*Received* 23 *May* 1983 *and in revised form* 20 *October* 1983)

NOMENCLATURE

c	specific heat of vapor
h	height of droplet above the plate
L	latent heat of vaporization
p	pressure
R	radius of droplet
T	temperature
T_s	temperature of droplet
T_w	plate temperature
T_∞	ambient temperature
$\mathbf{u}$	velocity vector.

Greek symbols

α	thermal diffusivity
λ	thermal conductivity
ν	kinematic viscosity
ρ	density
ψ	streamfunction.

Subscripts

n	normal component
t	tangential component.

1. INTRODUCTION

The phenomenon of a liquid droplet evaporating near a hot surface arises in many practical applications. For instance, in compression–ignition engines, gas turbines and other oil-fired appliances, fuel droplets are injected onto the hot walls of the combustion chamber where they must vaporize, prior to ignition. In the so-called open-chamber, stratified charge engines, the liquid fuel is directly injected into a hollow space in the piston, in the form of droplets or sprays. Evaporation of the droplets within this hollow space and subsequent combustion is known to improve engine performance.

Another situation involving vaporization of fuel droplets over a hot surface is that of fumigation. Here a small fraction of the fuel is deliberately injected onto the walls of the intake manifold. This fuel fraction evaporates and thereby reduces the delay in the formation of a combustible mixture and minimizes the rate of the pressure rise in the cylinder. Similar configurations may occur under fire fighting conditions where water droplets are injected to impinge on a hot ablating surface.

In view of the many applications, an understanding of the behavior of a liquid droplet evaporating near a hot surface is of fundamental importance. A systematic study of these phenomena began with the experimental observations of Tamura and Tanasawa [1] who presented a detailed description of the different modes of evaporation including the levitated spheroidal state. To be sure, similar observations were made much earlier by Johann Leidenfrost (1756) who reported dancing of water droplets over a red-hot iron spoon. Here a continuous vapor film separates the vaporizing liquid from the hot surface and the liquid droplet is suspended above the hot surface by its own vapor pressure. This phenomenon, known subsequently as the Leidenfrost phenomenon, is considered a special case of film boiling and is sometimes referred to as a small-scale vapor explosion [2].

There have been numerous attempts to give an analytical description of the Leidenfrost phenomenon. The first such attempt was due to Pleteneva and Rebinder [3] who presented a semianalytical solution to the problem. Among others, Gorton [4] postulated a model in which the vapor flow was assumed to be potential. Kistemaker [5] tried to improve on Gorton's work by dealing with the Navier–Stokes equations to describe the vapor flow below the droplet. Gottfried *et al.* [6] criticizes this solution (which is erroneous) and gives an overview of the previous attempts. Later, Satcunanathan [7] formulated a simple model to calculate the evaporation rate of a droplet next to a hot plate. Recently, Yang [8] has analyzed the evaporation process for the contact evaporation regime.